

# Modified Reconstruction of Standard Model in Non-Commutative Differential Geometry

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## Abstract

Sogami recently proposed the new idea to express Higgs particle as a kind of gauge particle by prescribing the generalized covariant derivative with gauge and Higgs fields operating on quark and lepton fields. The field strengths for both the gauge and Higgs fields are defined by the commutators of the covariant derivative by which he could obtain the Yang-Mills Higgs Lagrangian in the standard model. Inspired by Sogami's work, we present a modification of our previous scheme to formulate the spontaneously broken gauge theory in non-commutative geometry on the discrete space  $M_4 \times Z_2$  by introducing the generation mixing matrix  $K$  in  $d_\chi$  operation on the fields  $a_i(x, y)$  which compose the gauge and Higgs fields. The standard model is reconstructed according to the modified scheme, which does not yields not only any special relations between the particle masses but also the special restriction on the Higgs potential.

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## §1 Introduction

Many works have been done to realize the idea that the Higgs particle is a kind of the gauge field in non-commutative geometry(NCG) on the discrete space [1]~[10]. Sogami recently proposed the new idea [13] to express Higgs particle as a kind of gauge particle by prescribing the generalized covariant derivative with gauge and Higgs fields operating on quark and lepton fields. The field strengths for both the gauge and Higgs fields are defined by the commutators of the covariant derivative by which he could obtain the Yang-Mills Higgs Lagrangian in the standard model with the extra restriction on the coupling constant of the Higgs potential. The generation mixing is from the outset considered by setting up the interactions between fermions ( lepton, up and down quarks ) and the Higgs field with the Yukawa couplings in the matrix form. The coupling constant of the Higgs potential term is denoted by three Yukawa coupling constants which yields the relation of the Higgs and top quark masses  $M_H = \sqrt{2}m_t$  through the top quark mass dominance in the trace of the product of mass matrices.

The present authors have also developed the formalism [6], [7] to be applicable to the gauge theory with complex symmetry breaking pattern such as SU(5) GUT [7], [8] or SO(10) GUT [12]. However, the incorporation of generation mixing was not sufficient in our previous formalism though it was treated in the second reference of [6]. Inspired by th Sogami's work, we will in this paper introduce the generation mixing matrix  $K$  in the  $d_\chi$  operation on  $a_i(x, y)$  which composes the gauge and Higgs fields together with  $M(y)$  matrix to cause the spontaneous symmetry breakdown.  $K$  was originally introduced by Chamseddine et.al. [2] to keep the meaningful Higgs potential. In our formalism the Higgs potential is kept meaningful even if the generation mixing would not exist and so we here introduce  $K$  not to keep the Higgs potential but to obtain the realistic interactions between the quark and Higgs particle.

This paper is divided into four sections. The next section presents the modifications of our previous formalism based on the generalized differential calculus on  $M_4 \times Z_2$  so as to incorporate the generation mixing mechanism and color symmetry. In the second reference of [6],  $M_4 \times Z_3$  was necessary to take account of color symmetry responsible for strong interaction. However, by considering the generalized gauge field with direct product form we will only need  $M_4 \times Z_2$  in this paper. In this section a geometrical picture for the unification of the gauge and Higgs fields is realized, which is the ultimate understanding in this field. The third section is the application to the standard model which leads to the quite different predictions for particle masses from the Sogami's one. The last section is devoted to concluding remarks.

## §2 Generalized gauge field with direct product form

This section is mainly the review of our formulation to construct the gauge theory in non-commutative geometry on the discrete space [6] in which the extra discrete space  $Z_3$  was necessary to incorporate the strong interaction. We propose the modification to afford the direct product gauge group such as  $SU(3)_c \times SU(2)_L$  which enables us to use the discrete space  $M_4 \times Z_2$  in reconstructing the full standard model.

In addition the generation mixing matrix  $K$  is introduced to accord the Sogami's idea.  $K$  was initially considered in Ref.[2] to ensure the meaningful Higgs potential whereas our formulation did not need  $K$  to make the consistent gauge model with the spontaneous symmetry breakdown. However, we here introduce  $K$  to obtain the realistic Dirac Lagrangian with the generation mixing.

Let us first summarize the story of Ref.[6] though we modify it in such a way to include the generation mixing matrix  $K(y)$ . The generalized gauge field  $A(x, y)$  in non-commutative geometry on the discrete space  $M_4 \times Z_2$  was given as

$$A(x, y) = \sum_i a_i^\dagger(x, y) \mathbf{d}a_i(x, y), \quad (1)$$

where  $a_i(x, y)$  is the square-matrix-valued function and  $\mathbf{d}$  is the generalized exterior derivative defined as follows.

$$\begin{aligned} \mathbf{d}a_i(x, y) &= (d + d_\chi)a_i(x, y) = (d + d_\chi)a_i(x, y), \\ da_i(x, y) &= \partial_\mu a_i(x, y) dx^\mu, \\ d_\chi a_i(x, y) &= K(y)[-a_i(x, y)M(y) + M(y)a_i(x, -y)]\chi. \end{aligned} \quad (2)$$

Here  $dx^\mu$  is ordinary one form basis, taken to be dimensionless, in  $M_4$ , and  $\chi$  is the one form basis, assumed to be also dimensionless, in the discrete space  $Z_2$ . We have introduced  $x$ -independent matrix  $M(y)$  whose hermitian conjugation is given by  $M(y)^\dagger = M(-y)$ .  $K(y)$  is also assumed to be  $K(y)^\dagger = K(-y)$  and commutes with  $a_i(x, y)$  and  $M(y)$ . We here skip to explain detailed algebras with respect to non-commutative geometry because those are seen in Ref.[6]. According to Ref.[6], we can define the gauge fields  $A_\mu(x, y)$  and the Higgs field  $\Phi(x, y)$  as

$$\begin{aligned} A_\mu(x, y) &= \sum_i a_i^\dagger(x, y) \partial_\mu a_i(x, y), \\ \Phi(x, y) &= \sum_i a_i^\dagger(x, y) (-a_i(x, y)M(y) + M(y)a_i(x, -y)), \end{aligned} \quad (3)$$

with which Eq.(1) is rewritten as

$$A(x, y) = A_\mu(x, y) dx^\mu + K(y) \Phi(x, y) \chi. \quad (4)$$

In connection with  $K(y)$  in Eq.(2), it should be noticed that  $a_i(x, y)$  is also a representation in the generation space and so does  $A(x, y)$  in Eq.(4). Eq.(4) expresses the unified picture of the gauge and Higgs fields as the generalized connection on the discrete space  $M_4 \times Z_2$ .

We extend Eq.(1) to the generalized gauge field with the direct product form to incorporate gluon field on the same sheet as flavor gauge fields and to contain the generation mixing matrix  $K(y)$  to accord the Sogami's idea.

$$\mathcal{A}(x, y) = \sum_i a_i^\dagger(x, y) \mathbf{d}a_i(x, y) \otimes \mathbf{1} + \mathbf{1} \otimes \sum_j b_j^\dagger(x, y) \mathbf{d}b_j(x, y), \quad (5)$$

where the second term is responsible for the gluon field, so that actually  $\mathbf{d}b_j(x, y) = db_j(x, y)$  because the strong interaction does not break down spontaneously, and we denote  $b_j(x, y) = b_j(x)$  which means the strong interaction works on both discrete spaces ( $y = \pm$ ). In the same context as in Eq.(3), the gluon field  $G_\mu(x)$  is expressed as

$$G_\mu(x) = \sum_j b_j^\dagger(x) \partial_\mu b_j(x). \quad (6)$$

In order to identify  $A_\mu(x, y)$  and  $G_\mu(x)$  as true gauge fields, the following conditions have to be imposed.

$$\begin{aligned} \sum_i a_i^\dagger(x, y) a_i(x, y) &= 1, \\ \sum_j b_j^\dagger(x) b_j(x) &= \frac{1}{g_3}, \end{aligned} \quad (7)$$

where  $g_3$  is a constant related to the corresponding coupling constant as shown later.  $i$  and  $j$  are variables of the extra internal space which we can not now identify what they are. Eqs.(3) and (6) are very similar to the effective gauge field in Berry phase [11], which may lead to the identification of this internal space. In general, we can put the right hand side of the first equation in Eq.(7) to be  $1/g_y$ . However, we put it as it is to avoid the complexity.

Before constructing the gauge covariant field strength, we address the gauge transformation of  $a_i(x, y)$  and  $b_j(x)$  which is defined as

$$\begin{aligned} a_i^g(x, y) &= a_i(x, y) g(x, y), \\ b_j^g(x) &= b_j(x) g_3(x), \end{aligned} \quad (8)$$

where  $g(x, y)$  and  $g_3(x)$  are the gauge functions with respect to the corresponding flavor unitary group and the color  $SU(3)_c$  group, respectively. Then, we can get the gauge transformation of  $\mathcal{A}(x, y)$  to be

$$\begin{aligned} \mathcal{A}^g(x, y) &= g^{-1}(x, y) \otimes g_3^{-1}(x) \mathcal{A}(x, y) g(x, y) \otimes g_3(x) \\ &\quad + g^{-1}(x, y) \mathbf{d}g(x, y) \otimes \mathbf{1} + \mathbf{1} \otimes \frac{1}{g_3} g_3^{-1}(x) dg_3(x), \end{aligned} \quad (9)$$

where use has been made of Eq.(7) and as in Eq.(2),

$$\mathbf{d}g(x, y) = \partial_\mu g(x, y) dx^\mu + K(y)[-g(x, y)M(y) + M(y)g(x, y)]\chi. \quad (10)$$

Eq.(9) affords us to construct the gauge covariant field strength as follows:

$$\mathcal{F}(x, y) = F(x, y) \otimes \mathbf{1} + \mathbf{1} \otimes \mathcal{G}(x), \quad (11)$$

where  $F(x, y)$  and  $\mathcal{G}(x)$  are the field strengths of flavor and color gauge fields, respectively and given as

$$\begin{aligned} F(x, y) &= \mathbf{d}A(x, y) + A(x, y) \wedge A(x, y), \\ \mathcal{G}(x) &= dG(x) + g_3 G(x) \wedge G(x). \end{aligned} \quad (12)$$

The algebras of non-commutative differential geometry defined in Ref.[6] yields

$$\begin{aligned} F(x, y) &= \frac{1}{2} F_{\mu\nu}(x, y) dx^\mu \wedge dx^\nu \\ &\quad + K(y) D_\mu \Phi(x, y) dx^\mu \wedge \chi + K(y) K(-y) V(x, y) \chi \wedge \chi, \end{aligned} \quad (13)$$

where

$$\begin{aligned} F_{\mu\nu}(x, y) &= \partial_\mu A_\nu(x, y) - \partial_\nu A_\mu(x, y) + [A_\mu(x, y), A_\nu(x, y)], \\ D_\mu \Phi(x, y) &= \partial_\mu \Phi(x, y) + A_\mu(x, y)(M(y) + \Phi(x, y)) \\ &\quad - (\Phi(x, y) + M(y)) A_\mu(x, -y), \\ V(x, y) &= (\Phi(x, y) + M(y))(\Phi(x, -y) + M(-y)) - Y(x, y). \end{aligned} \quad (14)$$

$Y(x, y)$  in Eq.(14) is auxiliary field and expressed as

$$Y(x, y) = \sum_i a_i^\dagger(x, y) M(y) M(-y) a_i(x, y), \quad (15)$$

which may be independent or dependent of  $\Phi(x, y)$  and/or may be a constant field. If we define  $H(x, y) = \Phi(x, y) + M(y)$ , it is readily known that the function  $H(x, y)$  represents the unshifted Higgs field, whereas  $\Phi(x, y)$  denotes the shifted Higgs field with vanishing vacuum expectation value so that  $M(y)$  determines the scale and pattern of the spontaneous breakdown of gauge symmetry. In contrast to  $F(x, y)$ ,  $\mathcal{G}(x)$  is simply denoted as

$$\begin{aligned} \mathcal{G}(x) &= \frac{1}{2} G_{\mu\nu}(x) dx^\mu \wedge dx^\nu \\ &= \frac{1}{2} \{ \partial_\mu G_\nu(x) - \partial_\nu G_\mu(x) + g_3 [G_\mu(x), G_\nu(x)] \} dx^\mu \wedge dx^\nu. \end{aligned} \quad (16)$$

With the same metric structure on the discrete space  $M_4 \times Z_N$  as in Ref.[6] we can obtain the gauge invariant Yang-Mills-Higgs lagrangian (YMH)

$$\begin{aligned}
\mathcal{L}_{YMH}(x) &= -\text{Tr} \sum_{y=\pm} \frac{1}{g_y^2} < \mathcal{F}(x, y), \mathcal{F}(x, y) > \\
&= -\text{Tr} \sum_{y=\pm} \frac{1}{2g_y^2} F_{\mu\nu}^\dagger(x, y) F^{\mu\nu}(x, y) \\
&\quad + \text{Tr} \sum_{y=\pm} \frac{\alpha^2}{g_y^2} [K(-y)K(y)] (D_\mu \Phi(x, y))^\dagger D^\mu \Phi(x, y) \\
&\quad - \text{Tr} \sum_{y=\pm} \frac{\beta^4}{g_y^2} [K(-y)K(y)]^2 V^\dagger(x, y) V(x, y) \\
&\quad - \text{Tr} \sum_{y=\pm} \frac{1}{2g_y^2} G_{\mu\nu}^\dagger(x) G^{\mu\nu}(x),
\end{aligned} \tag{17}$$

where  $g_y$  is a constant relating to the coupling constant of the flavor gauge field and  $\text{Tr}$  denotes the trace over internal symmetry matrices including the color, flavor symmetries and generation space.  $\alpha$  and  $\beta$  emerge from the definition of metric  $< \chi, \chi > = -\alpha^2$  and  $< \chi \wedge \chi, \chi \wedge \chi > = \beta^4$ , respectively. The third term in the right hand side is the potential term of Higgs particle.

Let us turn to the fermion sector to construct the Dirac Lagrangian. This is also deeply indebted to Ref.[6] so that only main points should be explained by skipping details. Let us start to define the covariant derivative acting on the spinor field  $\psi(x, y)$  which is the representation of the corresponding semi simple group including  $\text{SU}(3)_c$ .

$$\mathcal{D}\psi(x, y) = (\mathbf{d} + \mathcal{A}(x, y))\psi(x, y), \tag{18}$$

which we call the covariant spinor one-form. The algebraic rules in Ref.[6] along with Eq.(5) leads Eq.(18) to

$$\begin{aligned}
\mathcal{D}\psi(x, y) &= \{ \mathbf{1} \otimes \mathbf{1} \partial_\mu dx^\mu + (A_\mu^f(x, y) \otimes \mathbf{1} dx^\mu + K(y)H(x, y) \otimes \mathbf{1} \chi) \\
&\quad + \mathbf{1} \otimes G_\mu^f(x) dx^\mu \} \psi(x, y),
\end{aligned} \tag{19}$$

where  $A_\mu^f(x, y)$  and  $G_\mu^f(x)$  are the differential representations with respect to  $\psi(x, y)$ . It should be noticed that  $\mathcal{D}\psi(x, y)$  is gauge covariant so that

$$\mathcal{D}\psi^g(x, y) = (g^f(x, y))^{-1} \otimes (g_s^f(x))^{-1} \mathcal{D}\psi(x, y), \tag{20}$$

where  $g^f(x, y) \otimes g_s^f(x)$  is the gauge transformation function with respect to the representation of  $\psi(x, y)$ . Corresponding with Eq.(18), the associated spinor one-form is introduced by

$$\tilde{\mathcal{D}}\psi(x, y) = \mathbf{1} \otimes \mathbf{1} \{ \gamma_\mu \psi(x, y) dx^\mu - i c_Y \psi(x, y) \chi \}, \tag{21}$$

where  $c_Y$  is a real dimensionless constant related to the Yukawa coupling constant between Higgs field and fermions. With the same inner products for spinor one-forms as in Ref.[6], we can get the Dirac Lagrangian.

$$\begin{aligned}\mathcal{L}_D(x, y) &= i\text{Tr} \langle \tilde{\mathcal{D}}\psi(x, y), \mathcal{D}\psi(x, y) \rangle \\ &= i\text{Tr} [\bar{\psi}(x, y)\gamma^\mu(\mathbf{1} \otimes \mathbf{1}\partial_\mu + A_\mu^f(x, y) \otimes \mathbf{1} + \mathbf{1} \otimes G_\mu^f(x))\psi(x, y) \\ &\quad + ic_Y\alpha^2\bar{\psi}(x, y) \sum_{y=\pm} K(y)H(x, y) \otimes \mathbf{1}\psi(x, y)],\end{aligned}\tag{22}$$

where  $\text{Tr}$  is also the trace over internal symmetry matrices including the color, flavor symmetries and generation space. The total Dirac Lagrangian is the sum over  $y$ :

$$\mathcal{L}_D(x) = \sum_{y=\pm} \mathcal{L}_D(x, y),\tag{23}$$

which is apparently invariant for the Lorentz and gauge transformations. Eqs.(17) and (23) along with Eq.(22) are crucially important to reconstruct the spontaneously broken gauge theory.

With these preparations, we can apply the direct product formalism proposed in this section to the standard model and compare it with the Sogami's presentation [13].

### §3 Model Construction

We first prescribe the fermion field  $\psi(x, y)$  in Eq.(18) with the existing leptons and quarks and then decide the generalized gauge field  $\mathcal{A}(x, y)$  in order to give the correct Dirac Lagrangian for the fermion sector in the standard model. Hereafter, the argument  $x$  is often abbreviated if no confusion.

$$\psi(x, +) = \begin{pmatrix} l_L \\ \gamma q_L \\ \sqrt{1-\gamma^2}q_L \end{pmatrix}, \quad \psi(x, -) = \begin{pmatrix} e_R \\ d_R \\ u_R \end{pmatrix},\tag{24}$$

where  $l_L$  and  $q_L$  are the left-handed doublet lepton and quark, respectively and  $\gamma$  is a constant necessary for the normalization of the kinetic term of  $q_L$ . It should be noticed that  $\psi(x, y)$  has the index for the three generation and so do the explicit expressions for fermions in the right hand sides of Eq.(24). For example, in the strict expressions  $e_R$ ,  $d_R$ , and  $u_R$  in the right of Eq.(24) should be written as

$$e_R \rightarrow \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}, \quad d_R \rightarrow \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix}, \quad u_R \rightarrow \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix},\tag{25}$$

respectively.

In order to obtain the Dirac Lagrangian for fermion fields in Eq.(24) we denote the generalized gauge field  $\mathcal{A}(x, y)$  in Eq.(5) as follows:

$$\mathcal{A}(x, y) = \mathcal{A}_\mu(x, y)dx^\mu \otimes \mathbf{1} + \Phi'(x, y)\chi \otimes \mathbf{1} + \mathbf{1} \otimes \mathcal{G}_\mu(x)dx^\mu. \quad (26)$$

$\mathcal{A}_\mu(x, \pm)$  are specified as

$$\mathcal{A}_\mu(x, +) = -\frac{i}{2} \sum_{k=1}^3 \tau^k A_\mu^k - \frac{i}{2} a \tau^0 B_\mu, \quad (27)$$

$$\mathcal{A}_\mu(x, -) = -\frac{i}{2} b B_\mu, \quad (28)$$

where  $A_\mu^i$  and  $B_\mu$  are SU(2) and U(1) gauge fields, respectively and so  $\tau^i$  is the Pauli matrices and  $\tau^0$  is  $2 \times 2$  unit matrix.  $a$  and  $b$  in Eqs.(27) and (28) are the U(1) hypercharge matrices corresponding to Eq.(24) and expressed as

$$a = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{3} & \\ 0 & 0 & \frac{1}{3} \end{pmatrix}, \quad b = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -\frac{2}{3} & \\ 0 & 0 & \frac{4}{3} \end{pmatrix}. \quad (29)$$

$\Phi'(x, y)$  is also written in accord with Eq.(24) as

$$\Phi'(x, y) = \begin{pmatrix} \Phi(x, y) & 0 & 0 \\ 0 & \Phi(x, y) & 0 \\ 0 & 0 & \tilde{\Phi}(x, y) \end{pmatrix}, \quad (30)$$

where

$$\Phi(x, +) = \Phi(x, -)^\dagger = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \tilde{\Phi}(x, +) = \tilde{\Phi}(x, -)^\dagger = i\tau^2 \Phi^*(x, +), \quad (31)$$

$$M(+) = M(-)^\dagger = \begin{pmatrix} 0 \\ \mu \end{pmatrix}, \quad \tilde{M}(+) = \tilde{M}(-)^\dagger = i\tau^2 M(+) = \begin{pmatrix} \mu \\ 0 \end{pmatrix}. \quad (32)$$

and  $\mathcal{G}_\mu(x)$  is expressed as

$$\mathcal{G}_\mu(x) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & G_\mu(x) & 0 \\ 0 & 0 & G_\mu(x) \end{pmatrix}, \quad (33)$$

with  $G_\mu(x)$  written as

$$G_\mu(x) = -\frac{i}{2} \sum_{a=1}^8 \lambda^a G_\mu^a, \quad (34)$$

where  $G_\mu^a$  is SU(3) color gauge field and so  $\lambda^a$  are Gell-Mann matrices.

With these specifications, the generalized gauge fields  $\mathcal{F}(x, y)$

$$\begin{aligned} \mathcal{F}(x, y) &= \frac{1}{2} \mathcal{F}_{\mu\nu}(x, y) \otimes \mathbf{1} dx^\mu \wedge dx^\nu + \mathcal{K}(y) D_\mu \mathcal{H}(x, y) \otimes \mathbf{1} dx^\mu \wedge \chi \\ &\quad + \mathcal{K}(y) \mathcal{K}(-y) \mathcal{V}(x, y) \otimes \mathbf{1} \chi \wedge \chi + \mathbf{1} \otimes \frac{1}{2} \mathcal{G}_{\mu\nu}(x) dx^\mu \wedge dx^\nu \end{aligned} \quad (35)$$



can be determined as

$$\mathcal{F}_{\mu\nu}(x, +) = -\frac{i}{2} \sum_{i=1}^3 \tau^i (\partial_\mu A_\nu^i - \partial_\nu A_\mu^i + \epsilon_{ijk} A_\mu^j A_\nu^k) - \frac{i}{2} \tau^0 a (\partial_\mu B_\nu - \partial_\nu B_\mu), \quad (36)$$

$$\mathcal{F}_{\mu\nu}(x, -) = -\frac{i}{2} \tau^0 b (\partial_\mu B_\nu - \partial_\nu B_\mu), \quad (37)$$

$$D_\mu \mathcal{H}(x, +) = (D_\mu \mathcal{H}(x, -))^\dagger = \begin{pmatrix} D_\mu H & 0 & 0 \\ 0 & D_\mu H & 0 \\ 0 & 0 & D_\mu \tilde{H} \end{pmatrix} \quad (38)$$

with

$$D_\mu H = \partial_\mu \Phi - \frac{i}{2} \sum_{i=1}^3 \tau^i A_\mu^i (\Phi + M) - \frac{i}{2} \tau^0 B_\mu (\Phi + M), \quad (39)$$

$$D_\mu \tilde{H} = \partial_\mu \tilde{\Phi} - \frac{i}{2} \sum_{i=1}^3 \tau^i A_\mu^i (\tilde{\Phi} + \tilde{M}) + \frac{i}{2} \tau^0 B_\mu (\tilde{\Phi} + \tilde{M}), \quad (40)$$

and

$$\mathcal{V}(x, y) = \begin{pmatrix} V(x, y) & 0 & 0 \\ 0 & V(x, y) & 0 \\ 0 & 0 & \tilde{V}(x, y) \end{pmatrix}, \quad (41)$$

with

$$V(x, +) = (\Phi + M)(\Phi^\dagger + M^\dagger) - Y(+), \quad V(x, -) = (\Phi + M)^\dagger (\Phi + M) - Y(-), \quad (42)$$

$$\tilde{V}(x, +) = (\tilde{\Phi} + \tilde{M})(\tilde{\Phi}^\dagger + \tilde{M}^\dagger) - \tilde{Y}(+), \quad \tilde{V}(x, -) = (\tilde{\Phi} + \tilde{M})^\dagger (\tilde{\Phi} + \tilde{M}) - \tilde{Y}(-), \quad (43)$$

and in addition to these expressions the generation mixing matrix  $\mathcal{K}(y)$  has the following form corresponding to Eq.(24).

$$\mathcal{K}(+) = \mathcal{K}(-)^\dagger = \mathcal{K} = \begin{pmatrix} K^l & 0 & 0 \\ 0 & K^d & 0 \\ 0 & 0 & K^u \end{pmatrix}, \quad (44)$$

where  $K^l$ ,  $K^d$  and  $K^u$  are mixing matrices for the lepton, down and up quarks, respectively.  $\mathcal{G}_{\mu\nu}(x)$  in Eq.(35) is given as

$$\mathcal{G}_{\mu\nu}(x) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & G_{\mu\nu} & 0 \\ 0 & 0 & G_{\mu\nu} \end{pmatrix}, \quad (45)$$

with  $G_{\mu\nu}$  in Eq.(16).

Putting Eq.(35) into Eq.(17) and rescaling gauge and Higgs fields we can obtain YMH for the standard model as follows:

$$\begin{aligned} \mathcal{L}_{YMH} = & -\frac{1}{4} \sum_{i=1}^3 (F_{\mu\nu}^i)^2 - \frac{1}{4} B_{\mu\nu}^2 \\ & + |D_\mu H|^2 - \lambda (H^\dagger H - \mu'^2)^2 \\ & - \frac{1}{4} \sum_{a=1}^8 G_{\mu\nu}^a {}^\dagger G^{a\mu\nu}, \end{aligned} \quad (46)$$

where

$$\begin{aligned}
F_{\mu\nu}^i &= \partial_\mu A_\nu^i - \partial_\nu A_\mu^i + g\epsilon^{ijk} A_\mu^j A_\nu^k, \\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\
D^\mu H &= \left[ \partial_\mu - \frac{i}{2} \left( \sum_i \tau^i g A_\mu^i + \tau^0 g' B_\mu \right) \right] (\Phi + M), \\
G_{\mu\nu}^a &= \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c,
\end{aligned} \tag{47}$$

with

$$g^2 = \frac{g_+^2}{21}, \quad g'^2 = \frac{g_+^2 g_-^2}{16g_+^2 + 5g_-^2}, \tag{48}$$

$$\lambda = \frac{g_+^4 g_-^2 \beta^4 \text{Tr}(\mathcal{K}\mathcal{K}^\dagger)^2}{\alpha^4 (g_+^2 + g_-^2)^2 (\text{Tr}(\mathcal{K}\mathcal{K}^\dagger))^2}, \quad \mu'^2 = \left( \frac{1}{g_+^2} + \frac{1}{g_-^2} \right) \text{Tr}(\mathcal{K}\mathcal{K}^\dagger) \alpha^2 \mu^2, \tag{49}$$

$$g_s^2 = \frac{g_+^2 g_-^2 g_3^2}{2(g_+^2 + g_-^2)}. \tag{50}$$

$\text{Tr}(\mathcal{K}\mathcal{K}^\dagger)$  and  $\text{Tr}(\mathcal{K}\mathcal{K}^\dagger)^2$  in above equations are given as

$$\begin{aligned}
\text{Tr}(\mathcal{K}\mathcal{K}^\dagger) &= \text{tr}(K^l K^{l\dagger}) + 3\text{tr}(K^d K^{d\dagger}) + 3\text{tr}(K^u K^{u\dagger}), \\
\text{Tr}(\mathcal{K}\mathcal{K}^\dagger)^2 &= \text{tr}(K^l K^{l\dagger})^2 + 3\text{tr}(K^d K^{d\dagger})^2 + 3\text{tr}(K^u K^{u\dagger})^2,
\end{aligned} \tag{51}$$

where  $\text{tr}$  is the trace over the generation space and the factor 3 comes from the trace of color indices. In deriving Eq.(46) the Higgs potential term  $V(x, +)$  is eliminated because the auxiliary field  $Y(x, +)$  is independent field owing to Eq.(32) whereas  $V(x, -)$  remains thanks to  $Y(x, -) = \mu^2$ . Eq.(48) yields the Weinberg angle with the parameter  $\delta = g_+/g_-$  to be

$$\sin^2 \theta_w = \frac{21}{16\delta^2 + 26}, \tag{52}$$

and Eq.(49) results in

$$m_w = \sqrt{\frac{1 + \delta^2}{2}} \left( \frac{\text{Tr}(\mathcal{K}\mathcal{K}^\dagger)}{21} \right)^{\frac{1}{2}} \alpha \mu, \tag{53}$$

$$m_H = \frac{2\delta\epsilon}{\sqrt{1 + \delta^2}} \left( \frac{\text{Tr}(\mathcal{K}\mathcal{K}^\dagger)^2}{\text{Tr}(\mathcal{K}\mathcal{K}^\dagger)} \right)^{\frac{1}{2}} \alpha \mu, \tag{54}$$

where  $\epsilon = \beta^2/\alpha^2$ .

These estimations are only valid in the classical level. Though we are tempted to compare them with the experimental values we will learn it to be impossible after getting the Dirac lagrangian in the fermion sector.

Let us turn to the construction of the Dirac Lagrangian for fermion sector. After the rescaling of the boson fields, we can write the covariant spinor one-form in Eq.(19) corresponding with the specification of Eq.(24) as

$$\begin{aligned}\mathcal{D}\psi(x, +) &= \mathbf{1} \otimes \mathbf{1} \partial_\mu dx^\mu + \left\{ -\frac{i}{2} \left( g \sum_{i=1}^3 \tau^i A_\mu^i + a g' \tau^0 B_\mu \right) \psi(x, +) dx^\mu \right. \\ &\quad \left. + \mathcal{K}(\Phi' + M') \psi(x, -) \chi \right\} \otimes \mathbf{1} - \mathbf{1} \otimes \frac{i}{2} \sum_{a=1}^8 \lambda^a g_c \mathcal{G}_\mu^a \psi(x, +) dx^\mu, \quad (55)\end{aligned}$$

and

$$\begin{aligned}\mathcal{D}\psi(x, -) &= \mathbf{1} \otimes \mathbf{1} \partial_\mu dx^\mu + \left\{ -\frac{i}{2} b g' B_\mu \psi(x, -) dx^\mu + \mathcal{K}^\dagger(\Phi' + M')^\dagger \psi(x, +) \chi \right\} \otimes \mathbf{1} \\ &\quad - \mathbf{1} \otimes \frac{i}{2} \sum_{a=1}^8 \lambda^a g_c \mathcal{G}_\mu^a \psi(x, -) dx^\mu. \quad (56)\end{aligned}$$

We can also express the associated spinor one-form in Eq.(21) as

$$\tilde{\mathcal{D}}\psi(x, \pm) = \mathbf{1} \otimes \mathbf{1} \{ \gamma_\mu \psi(x, \pm) dx^\mu - i c_Y \psi(x, \pm) \chi \}, \quad (57)$$

where corresponding to the expression Eq.(24)  $c_Y$  has the following form:

$$c_Y = \begin{pmatrix} c^l & 0 & 0 \\ 0 & c^d & 0 \\ 0 & 0 & c^u \end{pmatrix}. \quad (58)$$

$c^l$ ,  $c^d$  and  $c^u$  in Eq.(58) may be matrices in the generation space. According to Eqs.(22) and (23), we can get the Dirac lagrangian for the standard model as follows:

$$\begin{aligned}\mathcal{L}_D &= \sum_{y=\pm} i \langle \tilde{\mathcal{D}}\psi(x, y), \mathcal{D}\psi(x, y) \rangle \\ &= i \left( \bar{l}_L, \gamma \bar{q}_L, \sqrt{1 - \gamma^2} \bar{q}_L \right) \left\{ \partial_\mu - \frac{i}{2} \sum_{a=1}^8 \lambda^a g_c \mathcal{G}_\mu^a - \frac{i}{2} \sum_{i=1}^3 \tau^i g \mathcal{A}_\mu^i \right. \\ &\quad \left. - \frac{i}{2} \tau^0 \begin{pmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix} g' B_\mu \right\} \begin{pmatrix} l_L \\ \gamma q_L \\ \sqrt{1 - \gamma^2} q_L \end{pmatrix} \\ &\quad + i \left( \bar{e}_R, \bar{d}_R, \bar{u}_R \right) \left\{ \partial_\mu - \frac{i}{2} \sum_{a=1}^8 \lambda^a g_c \mathcal{G}_\mu^a - \frac{i}{2} \begin{pmatrix} -2 & 0 & 0 \\ 0 & -\frac{2}{3} & 0 \\ 0 & 0 & \frac{4}{3} \end{pmatrix} g' B_\mu \right\} \begin{pmatrix} e_R \\ d_R \\ u_R \end{pmatrix} \\ &\quad - \left( \bar{l}_L, \gamma \bar{q}_L, \sqrt{1 - \gamma^2} \bar{q}_L \right) g_Y (\Phi' + M') \begin{pmatrix} e_R \\ d_R \\ u_R \end{pmatrix} \\ &\quad - \left( \bar{e}_R, \bar{d}_R, \bar{u}_R \right) g_Y^\dagger (\Phi' + M')^\dagger \begin{pmatrix} l_L \\ \gamma q_L \\ \sqrt{1 - \gamma^2} q_L \end{pmatrix} \quad (59)\end{aligned}$$

which is sufficient as the Dirac Lagrangian of the standard model with the Yukawa coupling constants  $g_Y$  in matrix form given as

$$g_Y = c_Y \alpha^2 K = \begin{pmatrix} g_Y^l & 0 & 0 \\ 0 & g_Y^d & 0 \\ 0 & 0 & g_Y^u \end{pmatrix} = \begin{pmatrix} c^l \alpha^2 K^l & 0 & 0 \\ 0 & \gamma c^d \alpha^2 K^d & 0 \\ 0 & 0 & \sqrt{1 - \gamma^2} c^u \alpha^2 K^u \end{pmatrix}, \quad (60)$$

and yields the fermion mass as follows:

$$L_{\text{fermion mass}} = -\bar{e}_L M^l e_R - \bar{d}_L M^d d_R - \bar{u}_L M^u u_R - H.C., \quad (61)$$

where  $M^l = g_Y^l \mu'$ ,  $M^d = \gamma g_Y^d \mu'$ , and  $M^u = \sqrt{1 - \gamma^2} g_Y^u \mu'$  are the mass matrices appeared in Ref. [13]. In our case it seems to be impossible to connect the Higgs mass  $m_H$  and the top quark mass  $m_t$  because of the fact that the top quark contribution to  $\text{Tr}(\mathcal{K}\mathcal{K}^\dagger)^2$  and  $\text{Tr}(\mathcal{K}\mathcal{K}^\dagger)$  in Eq.(54) may not be necessarily dominant due to its trace form itself and so many unknown constants in Eq.(60) including  $c^l$ ,  $c^d$  and  $c^u$  which may be matrices in the generation space. Thus, we can say nothing about the relation of  $m_t$  and  $m_H$  written in Eq.(54). In addition, the parameter  $\epsilon = \beta^2/\alpha^2$  which is amount to the existence of the independent quadratic Higgs potential term of  $g_+$  and  $g_-$  appears in Eq(46). It makes the prediction of the Higgs mass completely ambiguous, and so it would be wise way to cease to make predictions about the relations between these particle masses though tempting.

## §4 Conclusion

Inspired by the Sogami's work [13] we introduced the generation mixing matrix  $\mathcal{K}$  in the  $d_\chi$  operation on  $a_i(x, y)$  which composes the gauge and the Higgs particles. The estimations of  $\text{Tr}(\mathcal{K}\mathcal{K}^\dagger)^2$  and  $\text{Tr}(\mathcal{K}\mathcal{K}^\dagger)$  in Eq(53) and Eq(54) are so difficult that we can not say anything about  $m_H$  and  $m_W$ .  $\mathcal{K}$  was originally introduced in [2] to prevent the Higgs potential terms from vanishing because of the auxiliary fields  $Y_{nm}$  and keep the meaningful Higgs potential terms, however it loses meaning in the case of one generation or no mixing between generations. Contrary to this we can get the previous results for  $m_H$  and  $m_W$  in [6] if  $\mathcal{K}$  is unit matrix in the corresponding spaces and so  $\text{Tr}(\mathcal{K}\mathcal{K}^\dagger)^2 = \text{Tr}(\mathcal{K}\mathcal{K}^\dagger) = 21$ . The expression of  $\sin^2 \theta_w$  in Eq.(52) considerably changes due to the implicit effects from the color and generation spaces, however it is not crucial difference to be exclusive from each other.

Contrary to ours, there is a simple relation  $m_H = \sqrt{2}m_t$  in Ref.[13]. This is because  $\lambda$  and  $m_H$  are in his paper expressed directly by the traces of Yukawa coupling constants not by  $K$  matrix appeared in our case. There seems considerable differences between our and Sogami's formalisms though both are based on the similar ideas to treat gauge and

the Higgs fields on the same footing. However, it is impossible to decide which case is better in the present time. There are many other attractive approaches in this field of non-commutative geometry. It is now expected that the Higgs search will be successful in the near future.

Talking about the quantization, we have the same number of parameters of the standard model in this paper as the ordinary one. Not only the special relations between parameters such as coupling constants but also the Higgs potentials with the restrictive forms are never introduced here. Thus, the quantization can be performed in the same way as in the ordinary one.

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